BIRZEIT UNIVERSITY

Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Engineering Probability and Statistics ENEE 2307

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Midterm Exam

First Semester 2017-2018

Date: Sunday 12/11/2017 — Time: 75 minutes

Name: — Student #:

Opening Remarks:

- This is a 75-minute exam. Calculators are allowed, but books, notes, formula sheets, and other aids are not allowed.
- You are required to show all your work and provide the necessary explanations everywhere to get full credit.

Problem 1: 16 Points

Let A, B, and C be three independent events defined over a sample space S such that P(A) = ½, P(B) = 1/3, and P(C) = 1/4.

a. Find $P(A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Independent events

$$P(A \cup B) = P(A) + P(B) - P(A)P(B)$$
$$= 0.5 + 1/3 - 1/6 = 2/3$$

b. Find $P(B|C)$

$$P(B/C) = P(B) = 1/3$$

Or

$$P(B/C) = \frac{P(B \cap C)}{P(C)} = \frac{1/3 * 1/4}{1/4} = 1/3$$

c. Find the probability that none of the events will occur

$$P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\overline{A \cup B \cup C}) = 1 - P(A \cup B \cup C)$$
$$= 1 - [1/2 + 1/3 + 1/4 - 1/2 * 1/3 - 1/2 * 1/3 - 1/2 * 1/4 - 1/3 * 1/4 + 1/2 * 1/3 * 1/4] = 1 - \frac{3}{4} = \frac{1}{4}$$

Find the probability that all three events occur

$$P(A \cap B \cap C) = 1/2 * 1/3 * 1/4 = 1/24$$

Problem 2: 18 Points

The velocity V of an object is a continuous random variable with the following probability density function

$$f_V(v) = \begin{cases} kv(1-v), & 0 \le v \le 1 \\ 0, & otherwise \end{cases}$$

a. Find k so that $f_V(v)$ is a valid probability density function

$$1=\int_0^1 f(x)dx=\int_0^1 \mathrm{kv}(1-\mathrm{v})dv=\mathrm{k}(\frac{\mathrm{v}^2}{2}-\frac{\mathrm{v}^3}{3})\Big]_0^1=k\left(\frac{1}{2}-\frac{1}{3}-0\right)=\frac{k}{6}$$

$$k=6$$

b. Find the mean value of the velocity (leave your answer in terms of k)

$$\mu_v=E(v)=\int_{-\infty}^{\infty} vf_v(v)dv=\int_0^1 kv^2(1-v)dv=\mathrm{k}(\frac{\mathrm{v}^3}{3}-\frac{\mathrm{v}^4}{4})\Big]_0^1=\frac{1}{12}$$

c. Find $P(V\leq\frac{a}{2})$ (leave your answer in terms of k)

$$=\int_0^{a/2} f_v(v)dv=\mathrm{k}(\frac{\mathrm{v}^2}{2}-\frac{\mathrm{v}^3}{3})\Big]_0^{a/2}=k\frac{a}{12}$$

Problem 3: 16 Points

Computer packets that arrive at a certain router follow the Poisson process with a mean rate of 5 packets/second

a. Find the probability that three packets arrive in one second

$$P(X\leq 3)=(\lambda)^x\frac{e^{-\lambda}}{x!}=(5)^3\frac{e^{-5}}{3!}=0.14037$$

b. Find the probability that at least two packets arrive in two seconds.

$$P(X\geq 2)=1-P(X<2)=1-(5*2)^0\frac{e^{-5*2}}{0!}-(5*2)^1\frac{e^{-5*2}}{1!}=1-11e^{-10}=0.9995$$

c. Find the mean value of packets that arrive in two seconds.

$$\mu_x=E\{X\}=\lambda T=2*5=10$$

Problem 4: 18 Points

The monthly income of Ahmed is a Gaussian random variable X with mean NIS 3000 and standard deviation NIS 400 and is independent from month to month

a. Find the probability that Ahmad's income in a given month is greater than NIS 3500

$$\mathrm{P(X>3500)=1\text{-}P(X}\leq 3500)=1-\Phi\left(\frac{3500-3000}{400}\right)=0.1056$$

b. Find the probability that his income in a given month is between NIS 2600 and 3400

$$\mathrm{P(2600<X<3400)}=\Phi\left(\frac{3400-3000}{400}\right)-\Phi\left(\frac{2600-3000}{400}\right)$$

$$2\Phi(1)-1=2*0.8413=0.6826$$

c. Find the probability that his income in two consecutive months is greater than NIS 3000 in each one of them.

Each month greater than NIS 3000

$$\mathrm{P(X>3000)=1\text{-}P(X}\leq 3000)=1-\Phi\left(\frac{3000-3000}{400}\right)=0.5$$

Two consecutive months is greater than NIS 3000 in each one of them.

P(X>3000)∩P(X>3000)=P(X>3000)P(X>3000)=0.5*0.5=0.25

Since all months are independent.

Problem 5: 16 Points

A test consists of 20 multiple-choice questions (MCQs) with every question having four options, out of which only one is correct. A student passes the test if he answers 18 or more questions correctly. If an unprepared student only guesses the correct answer out of the four possible choices,

a. Find the probability that the student passes the exam

$$P(X\geq 18)=P(X>270)=\sum_{x=18}^{20}\binom{20}{x}\left(\frac{1}{4}\right)^{x}*\left(1-\frac{1}{4}\right)^{20-x}$$

$$\binom{20}{18}\left(\frac{1}{4}\right)^{18}*\left(1-\frac{1}{4}\right)^{20-18}+\binom{20}{19}\left(\frac{1}{4}\right)^{19}*\left(1-\frac{1}{4}\right)^{1}+\binom{20}{20}\left(\frac{1}{4}\right)^{20}*\left(1-\frac{1}{4}\right)^{0}$$

$$=1.61*10^{-9}$$

b. Find the average number of correct questions that he answers

$$\mu_x=E[X]=np=20*\frac{1}{4}=5$$

Problem 6: 16 Points

It is known that 20% of computers in a certain computer lab are infected (I) with a virus. If infected, a computer requires formatting (F) with probability 0.9, while if not infected (NI) it requires formatting with probability 0.25.

a. Find the probability that a randomly chosen computer requires formatting.

$$\mathrm{P}(F)=P(I)P(F/I)+P(NI)P(F/NI)=0.2*0.9+0.8*0.25=0.38$$

b. If a randomly selected computer is formatted, what the probability that it is infected with the virus?

$$P(I/F)=\frac{P(I\cap F)}{P(F)}=\frac{P(I)P(F/I)}{P(F)}=\frac{0.2*0.9}{0.38}=0.4736$$

Good Luck